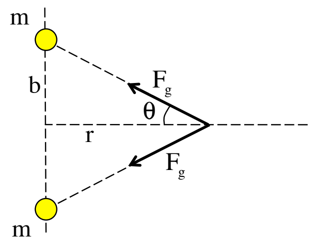
Problem 13.26

Determine the gravitational field "r" units down the *x-axis* due to the two masses shown.



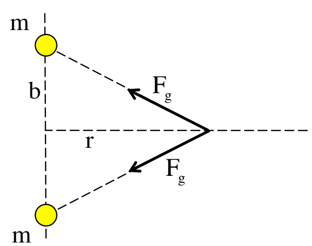
Because the masses are the same, and due to the symmetry of the situation, the magnitude and angle of the accelerations acting at "r" will be the same. As such, the *y-components* will add to zero and all we will have to calculate is the *x-component* of one, then double to accommodate the second. That is:

$$a_x = 2\left(\frac{Gm}{\left(\left(b^2+r^2\right)^{1/2}\right)^2}\right)\cos\theta$$

$$= 2\left(\frac{Gm}{\left(\left(b^2+r^2\right)^{1/2}\right)^2}\right)\left(\frac{r}{\left(b^2+r^2\right)^{1/2}}\right)$$

$$= 2\frac{Gm}{\left(\left(b^2+r^2\right)^{3/2}\right)}r$$

b.) Why should the net field go to zero as "r" goes to zero?



As "r" goes to zero, the *x-components* of the two forces becomes less and less until they are zero, and the *y-components* always add to zero as they are equal and opposite in direction.

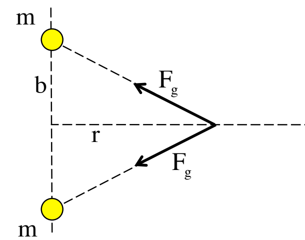
c.) Prove *Part b's* answer mathematically.

We derived the acceleration in the *x-direction*, as shown below.

$$a_x = 2\left(\frac{Gm}{\left(\left(b^2+r^2\right)^{1/2}\right)^2}\right)\cos\theta$$

As $r \Rightarrow 0,\ \theta \Rightarrow 90^\circ$ and $\cos 90^\circ = 0.$

d.) If we let "r" go to infinity, what happens?



As you get farther and farther away, the system begins to resemble more and more that of a point mass of mass "2m." With that, the field should be:

$$a_x = \frac{G(2m)}{r^2}$$

e.) prove your response to *Part d.*

If you note that as "r" gets really big, the contribution of "b" in the expression gets less and less significant and essentially becomes nil, and we can write:

$$a_x = 2\frac{Gm}{\left(\cancel{b^2}+r^2\right)^{3/2}}r$$

$$= 2\frac{Gm}{\left(r^2\right)^{3/2}}r$$

$$= 2\frac{Gm}{r^2}$$